Practica 7

Aproximaciones, Taylor.

Problema 1

En los siguientes casos use la formula de Taylor para f(x,y) en el origen para encontrar una aproximación cuadratica y cubica de la función en el punto.

a) $f(x,y) = x\,e^y$

b) $f(x,y) = e^x \cos(y)$

c) $f(x,y) = e^x \ln(1+y)$

Ejercicio 1

a)

$$f(x, y) = xe^y \;\Rightarrow\; f_x = e^y,\ f_y = xe^y,\ f_{xx} = 0,\ f_{xy} = e^y,\ f_{yy} = xe^y$$

$$f(x, y) \approx f(0, 0) + xf_x(0, 0) + yf_y(0, 0) + \frac{1}{2}\left[x^2 f_{xx}(0, 0) + 2xyf_{xy}(0, 0) + y^2 f_{yy}(0, 0)\right]$$

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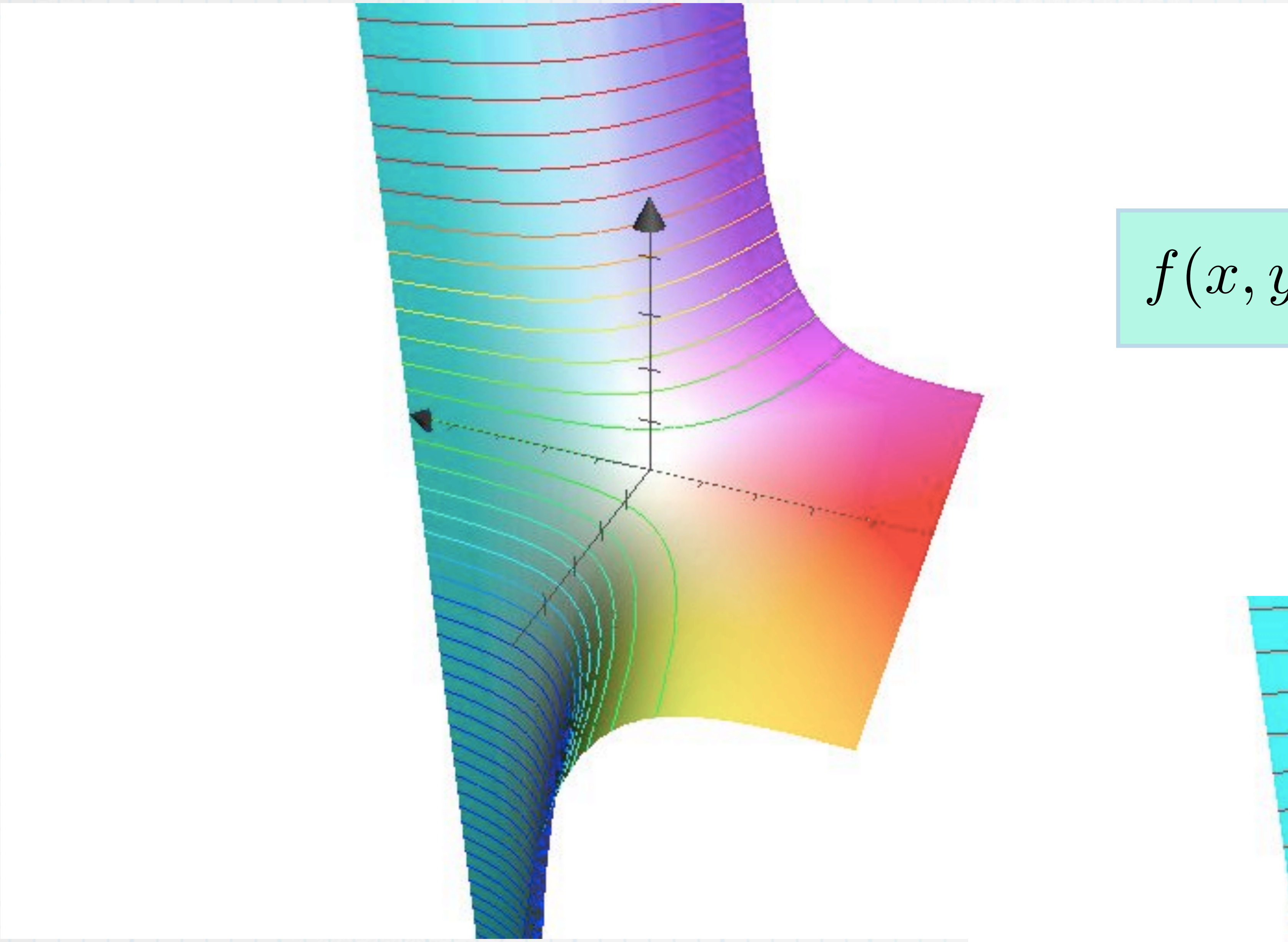
$$f(x,y) \approx f(0,0) + xf_x(0,0) + yf_y(0,0) + \frac{1}{2}\left[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$

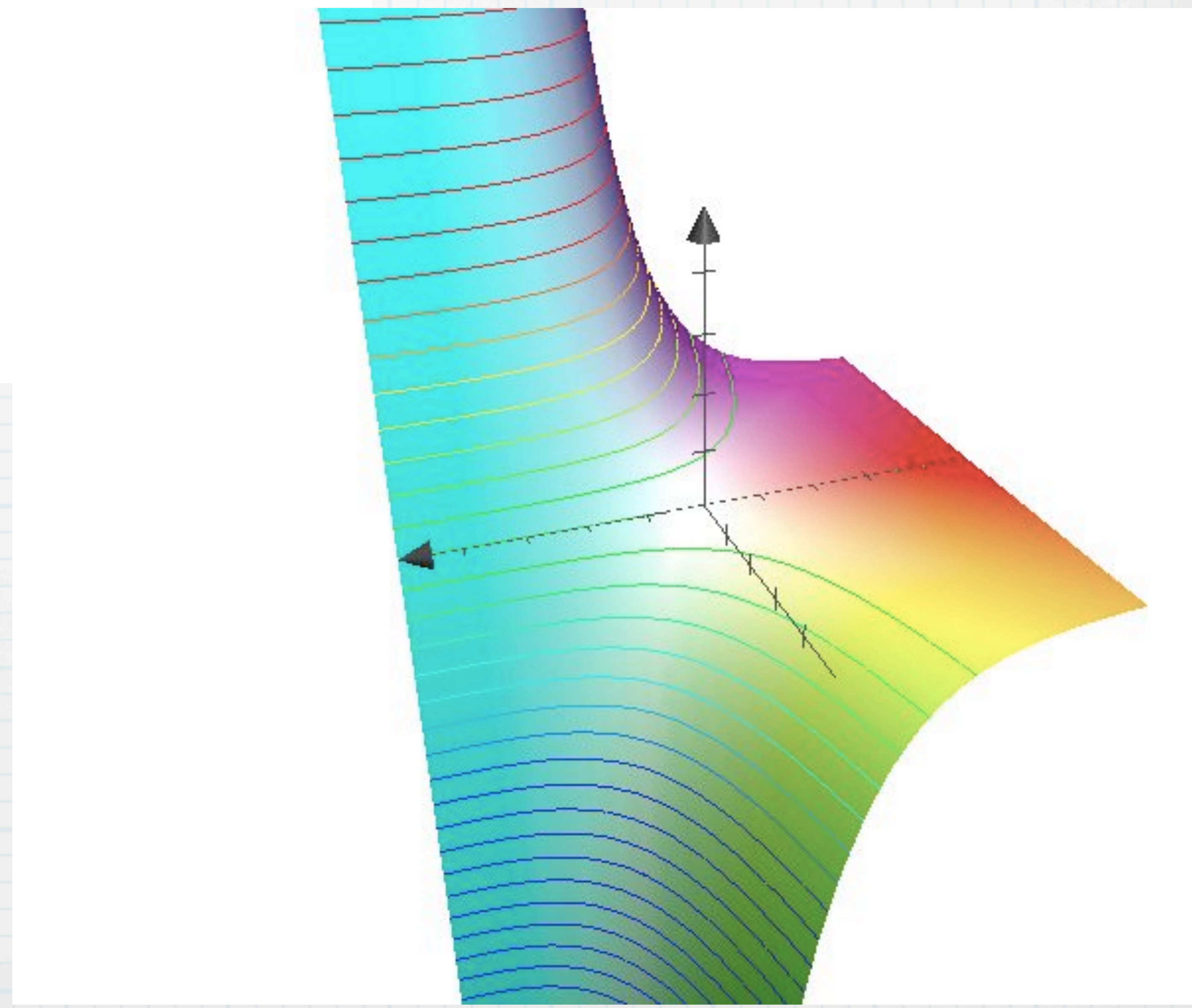
$$= 0 + x\cdot 1 + y\cdot 0 + \frac{1}{2}\left(x^2\cdot 0 + 2xy\cdot 1 + y^2\cdot 0\right)$$

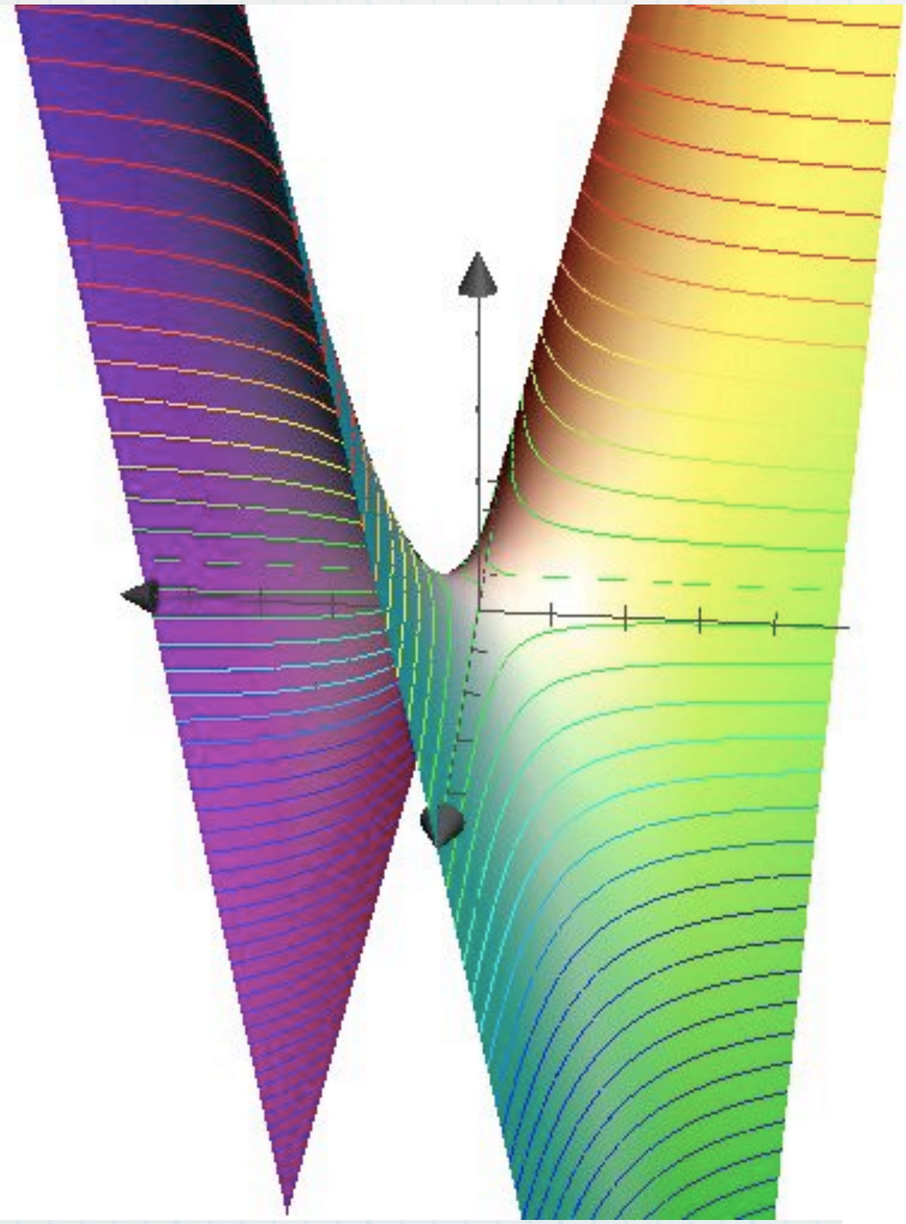
$$= x + xy$$

Aproximación cuadratica

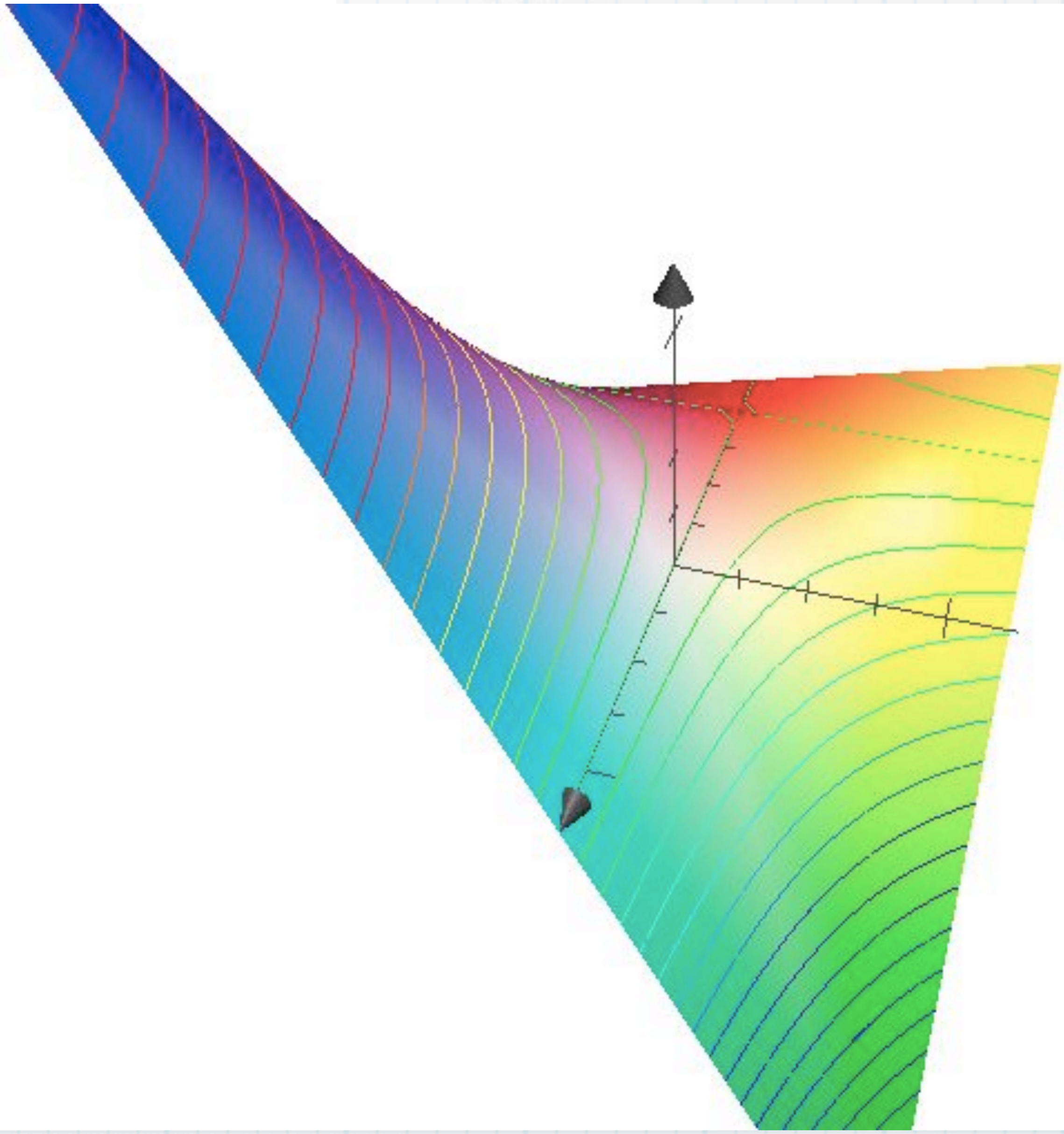


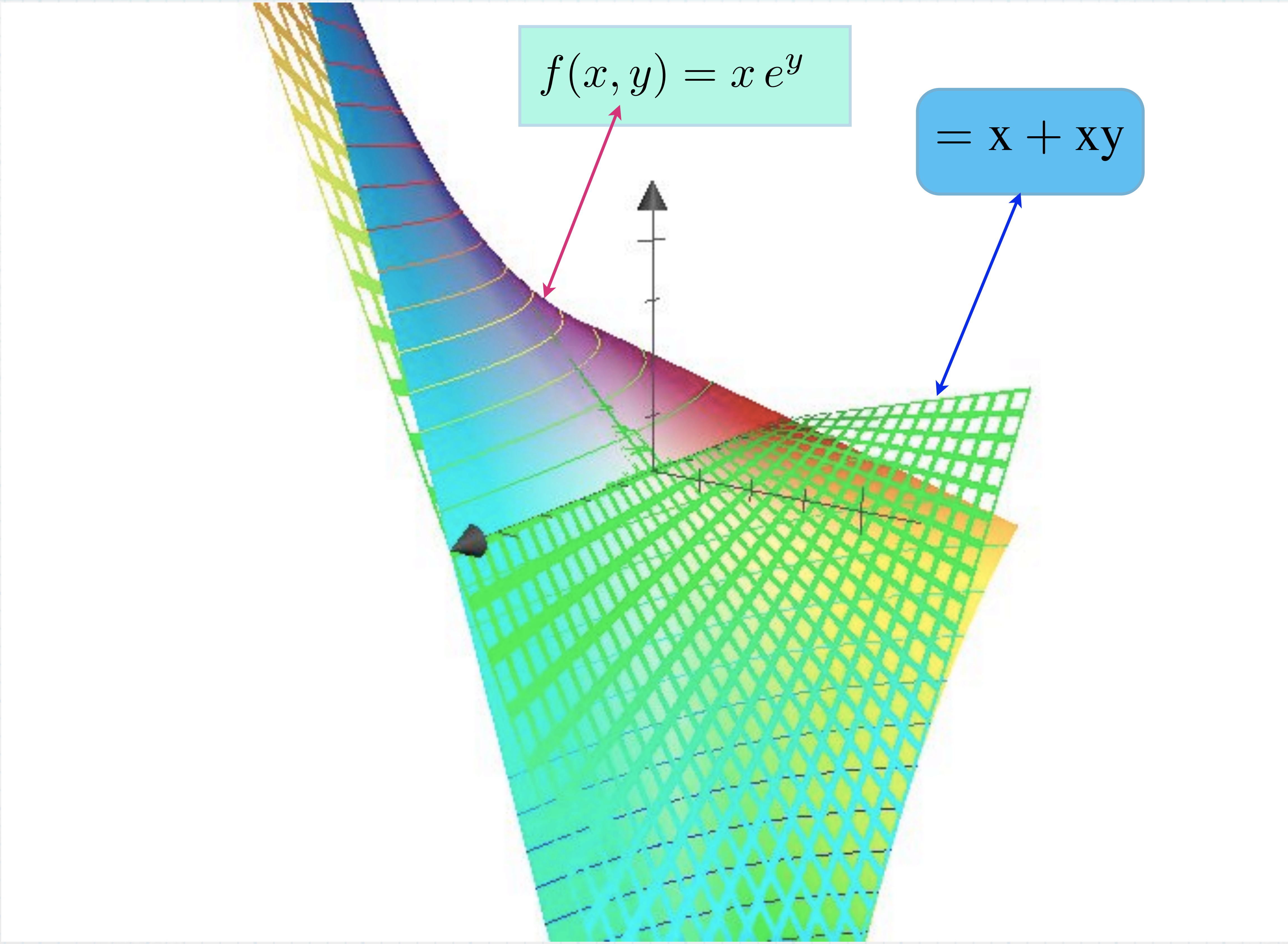
$$f(x, y) = x\, e^y$$





$= x + xy$




$f(x,y) = x\,e^y$
$= \mathbf{x + xy}$

$$f_{xxx} = 0,\ f_{xxy} = 0,\ f_{xyy} = e^y,\ f_{yyy} = xe^y$$

$$f(x,y) \approx f(0,0) + xf_x(0,0) + yf_y(0,0) + \frac{1}{2}\left[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$

$$+ \frac{1}{6}\left[x^3 f_{xxx}(0,0) + 3x^2 y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0) + y^3 f_{yyy}(0,0)\right]$$

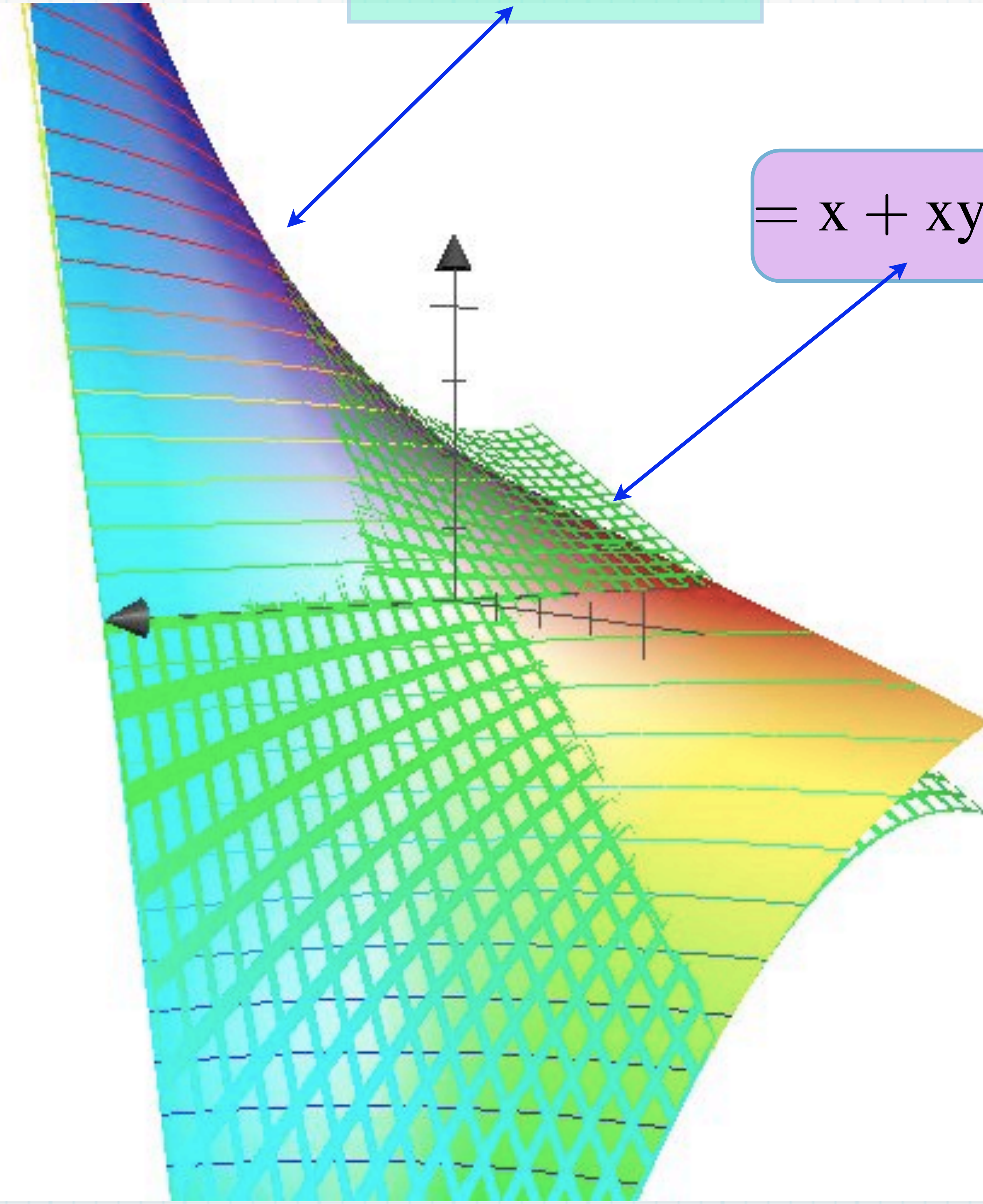
$$= x + xy + \frac{1}{6}\left(x^3 \cdot 0 + 3x^2 y \cdot 0 + 3xy^2 \cdot 1 + y^3 \cdot 0\right)$$

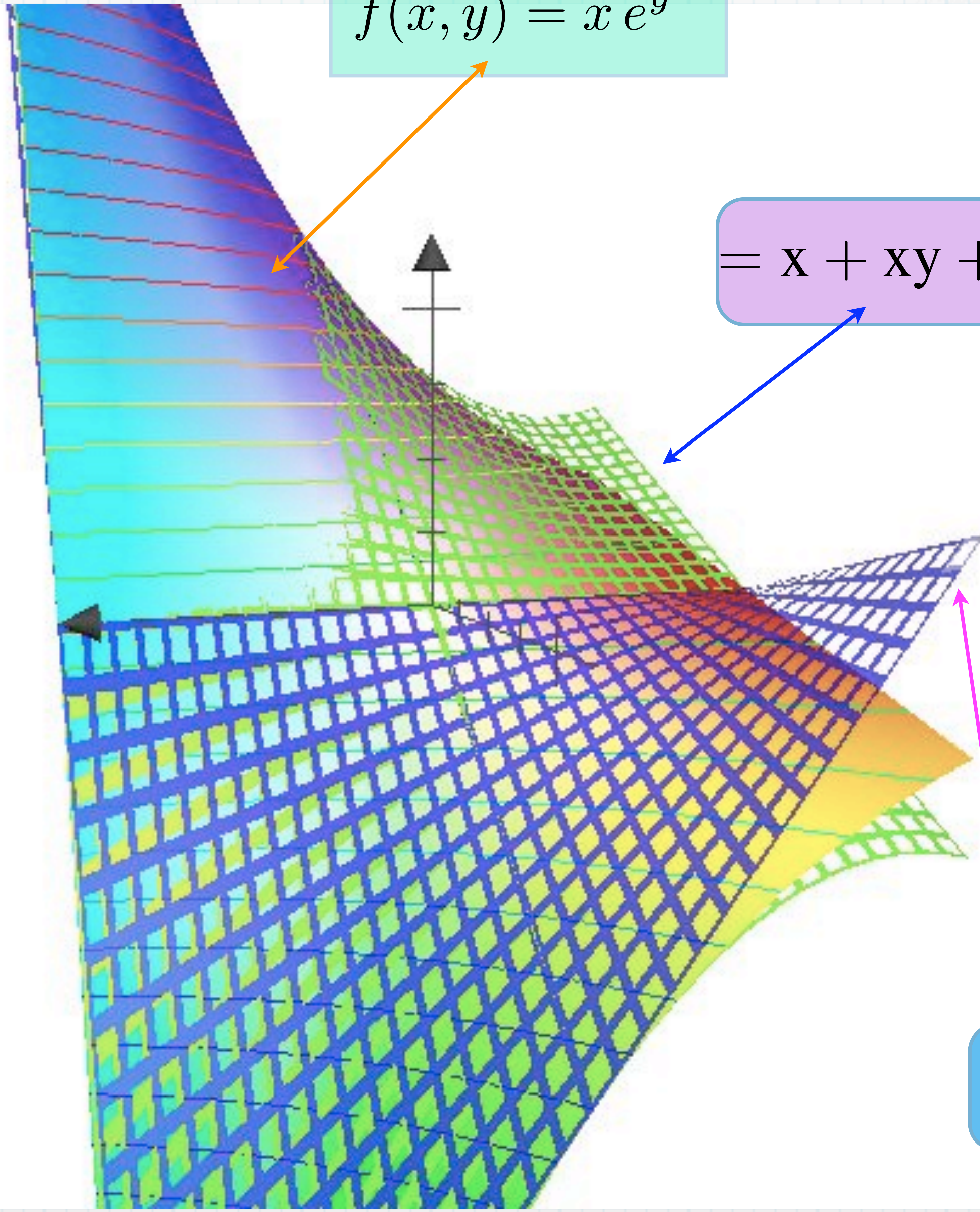
$$= x + xy + \frac{1}{2}xy^2$$

Aproximación cubica

$$f(x, y) = x\,e^{y}$$

$$= \mathrm{x} + \mathrm{xy} + \tfrac{1}{2}\,\mathrm{xy}^2$$





$f(x,y) = x\,e^y$

$= \mathrm{x} + \mathrm{xy} + \frac{1}{2}\,\mathrm{xy}^2$

$= \mathrm{x} + \mathrm{xy}$

b) $f(x, y) = e^x \cos y \;\Rightarrow\; f_x = e^x \cos y,\, f_y = -e^x \sin y,\, f_{xx} = e^x \cos y,$

$$f_{xy} = -e^x \sin y,\, f_{yy} = -e^x \cos y$$

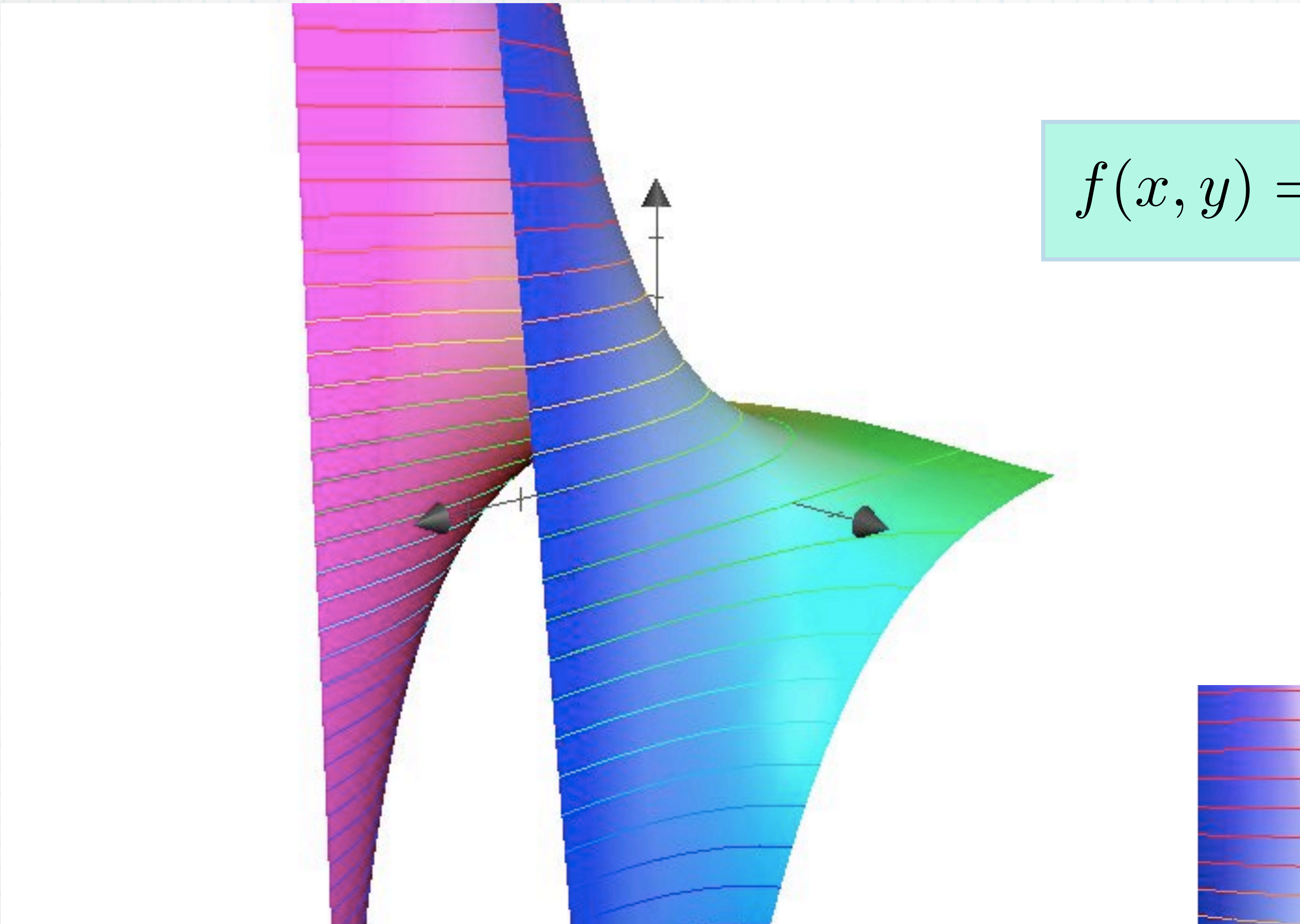
$$\Rightarrow\; f(x, y) \approx f(0,0) + x f_x(0,0) + y f_y(0,0) + \frac{1}{2}\left[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$

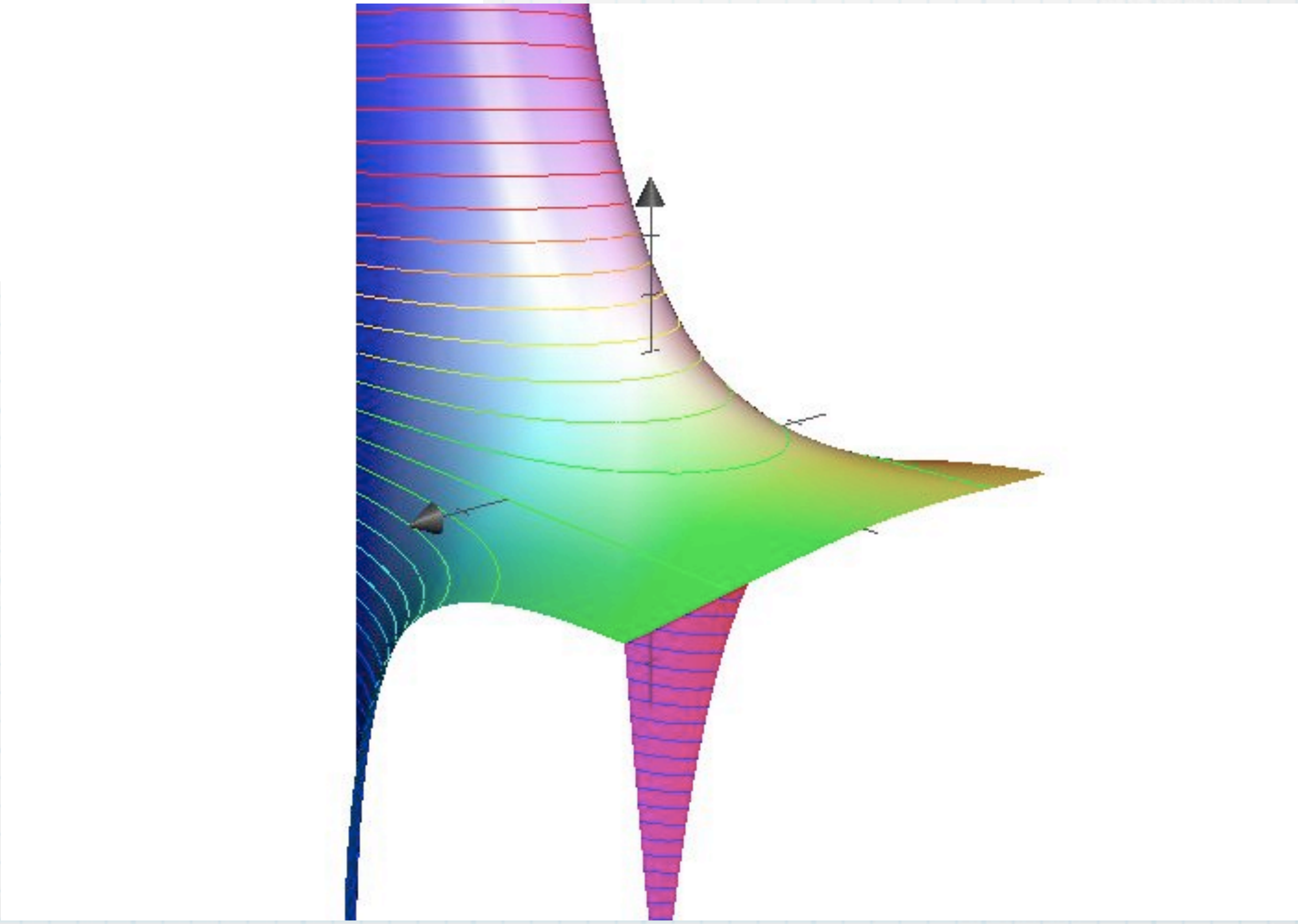
$$= 1 + x \cdot 1 + y \cdot 0 + \frac{1}{2}\left[x^2 \cdot 1 + 2xy \cdot 0 + y^2 \cdot (-1)\right]$$

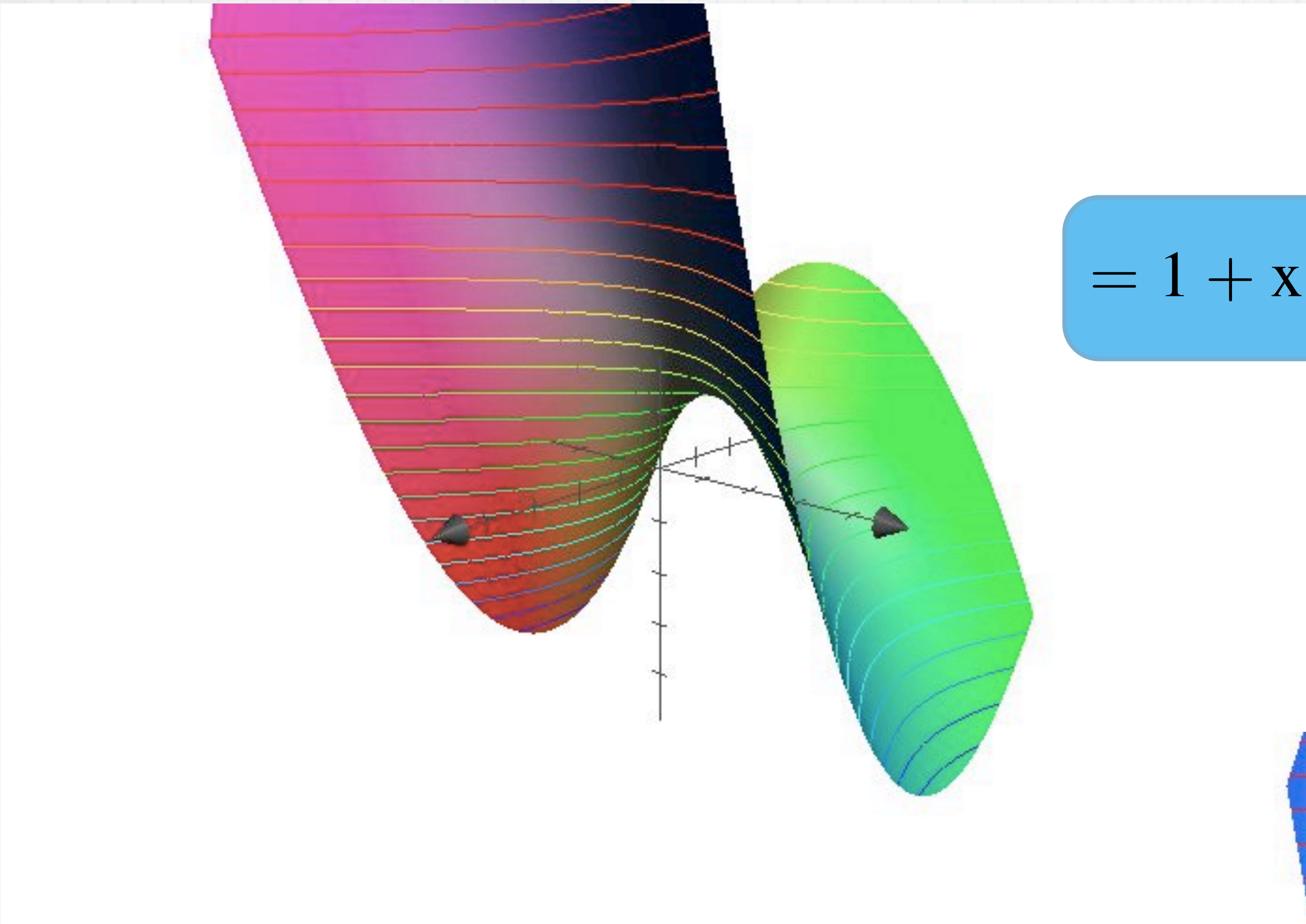
$$= 1 + x + \frac{1}{2}\left(x^2 - y^2\right)$$

Aproximación cuadratica

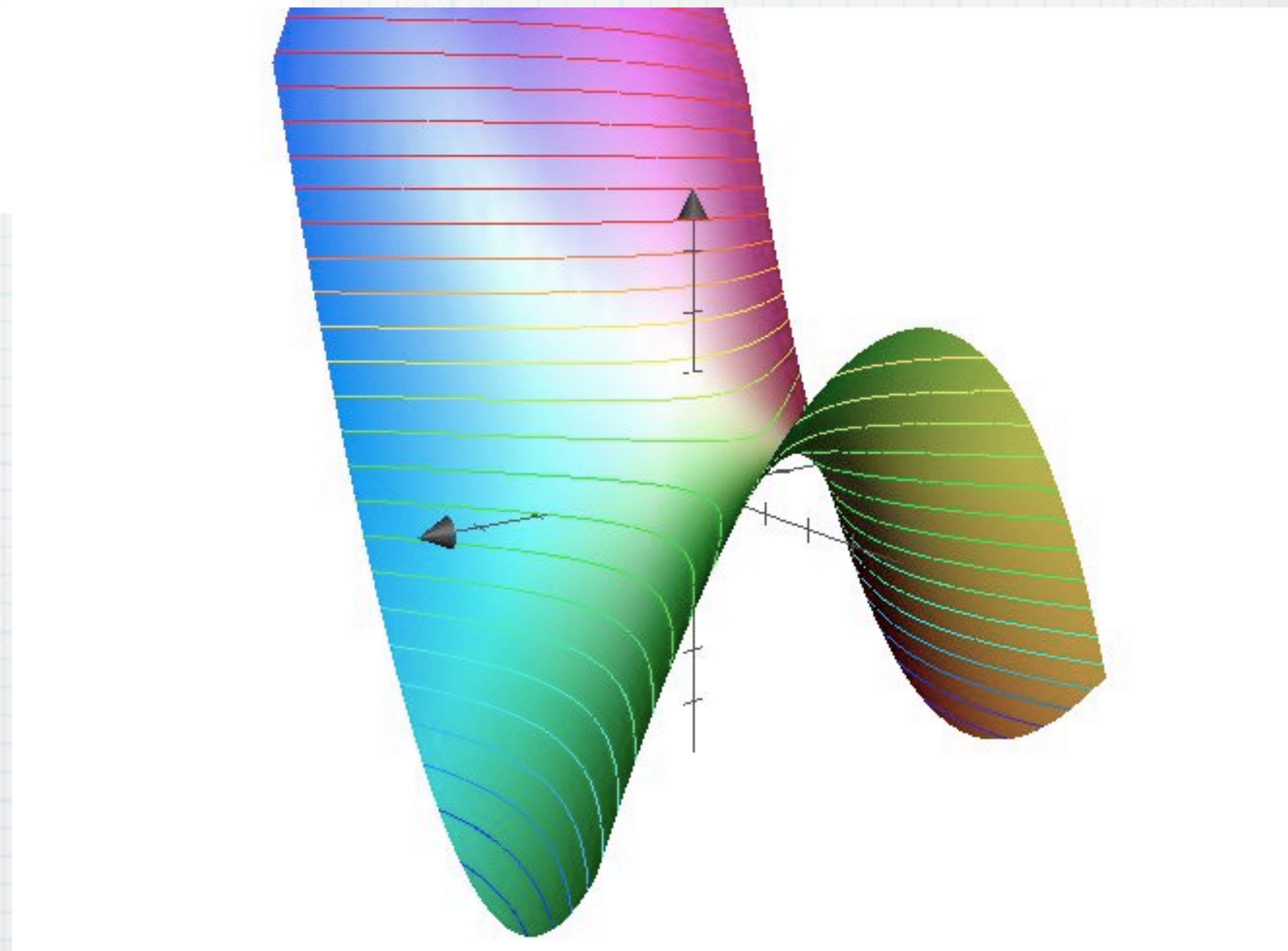
$$f(x, y) = e^x \cos(y)$$

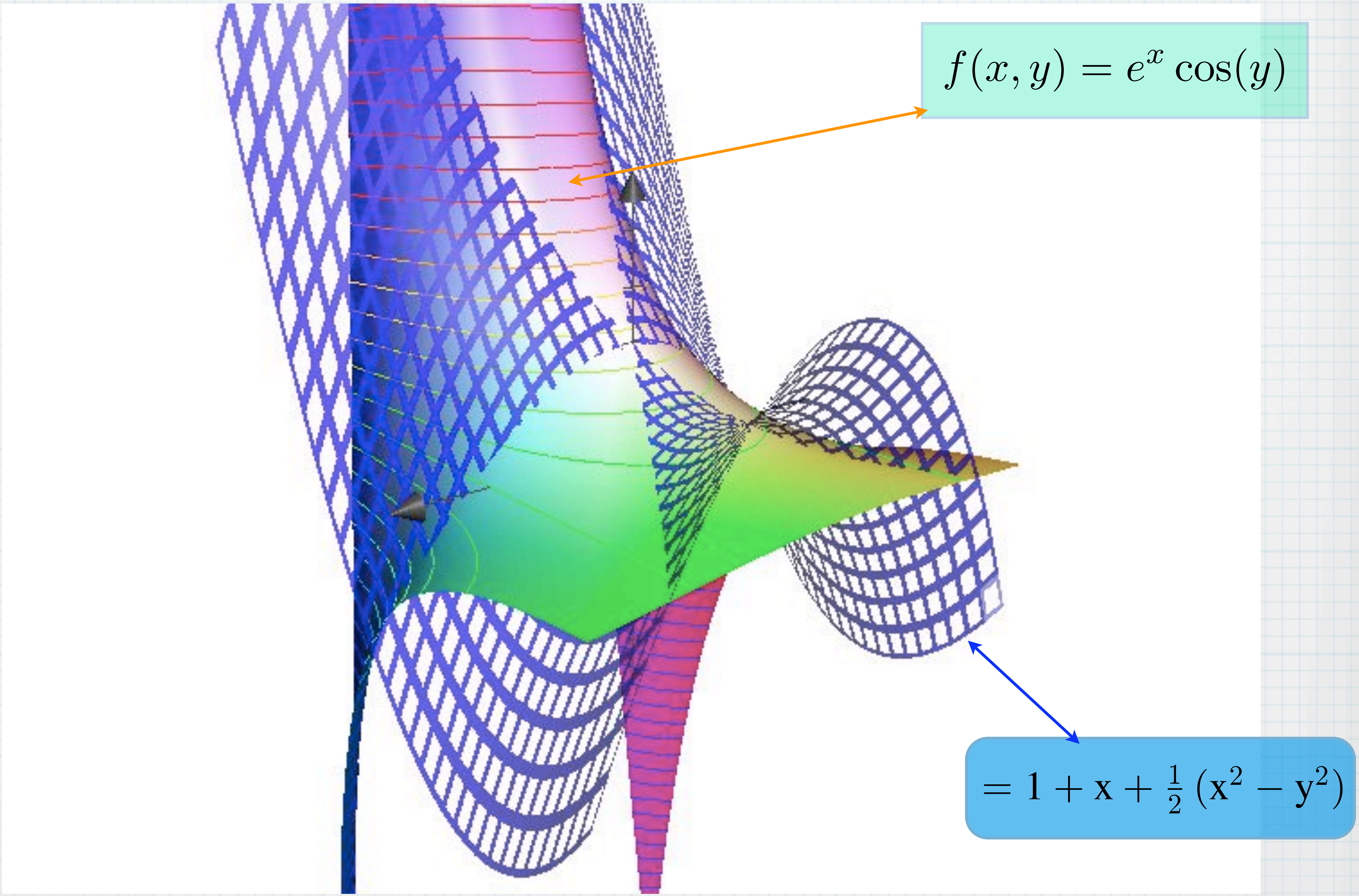






$$= 1 + x + \frac{1}{2}\left(x^2 - y^2\right)$$




$f(x,y) = e^x \cos(y)$
$= 1 + x + \frac{1}{2}(x^2 - y^2)$

$$f_{xxx} = e^x \cos y,\ f_{xxy} = -e^x \sin y,\ f_{xyy} = -e^x \cos y,\ f_{yyy} = e^x \sin y$$

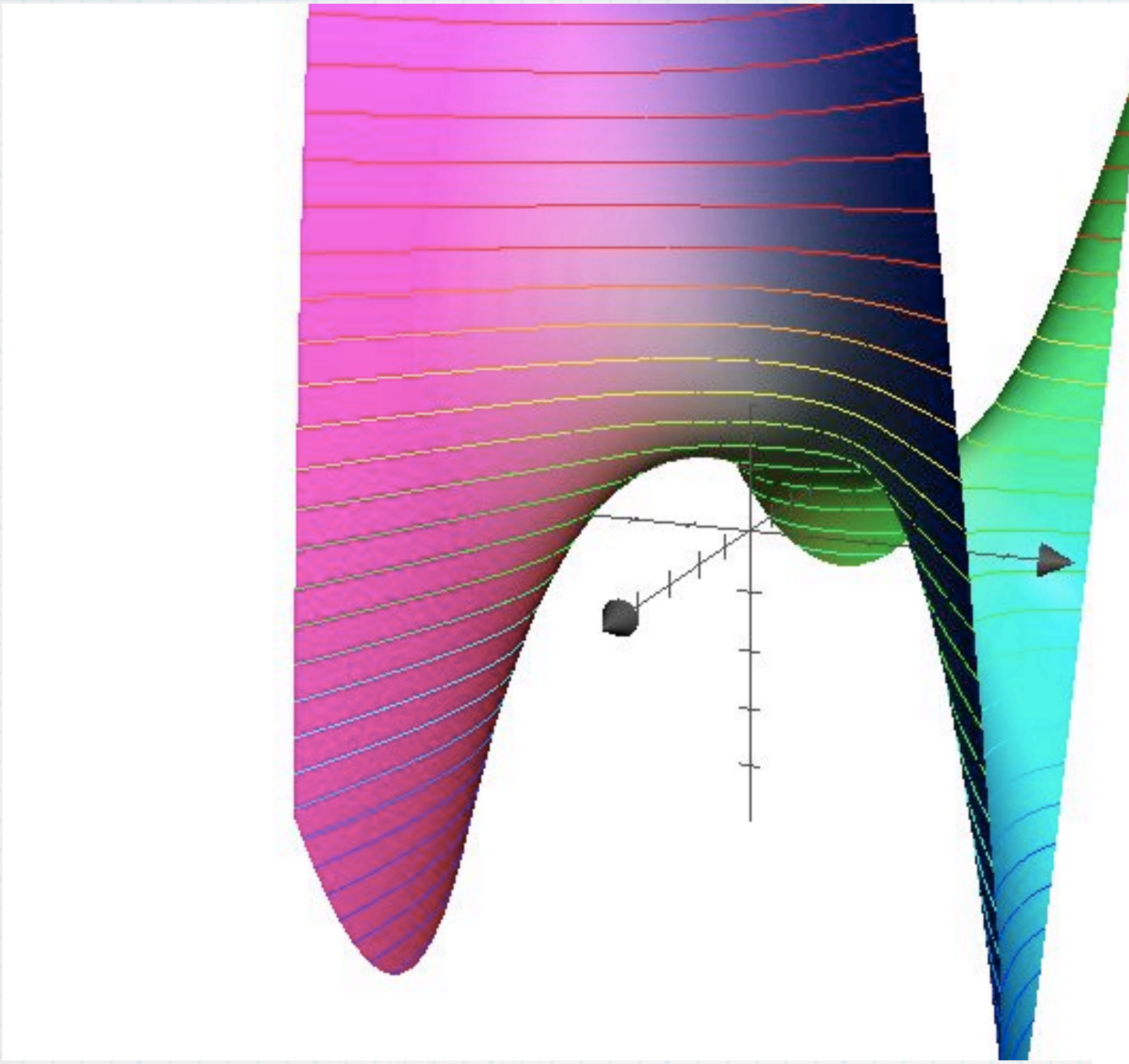
$$\Rightarrow\ f(x,y) \approx f(0,0) + xf_x(0,0) + yf_y(0,0) + \frac{1}{2}\left[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$

$$+\ \frac{1}{6}\left[x^3 f_{xxx}(0,0) + 3x^2 y f_{xxy}(0,0) + 3xy^2 f_{xyy}(0,0) + y^3 f_{yyy}(0,0)\right]$$

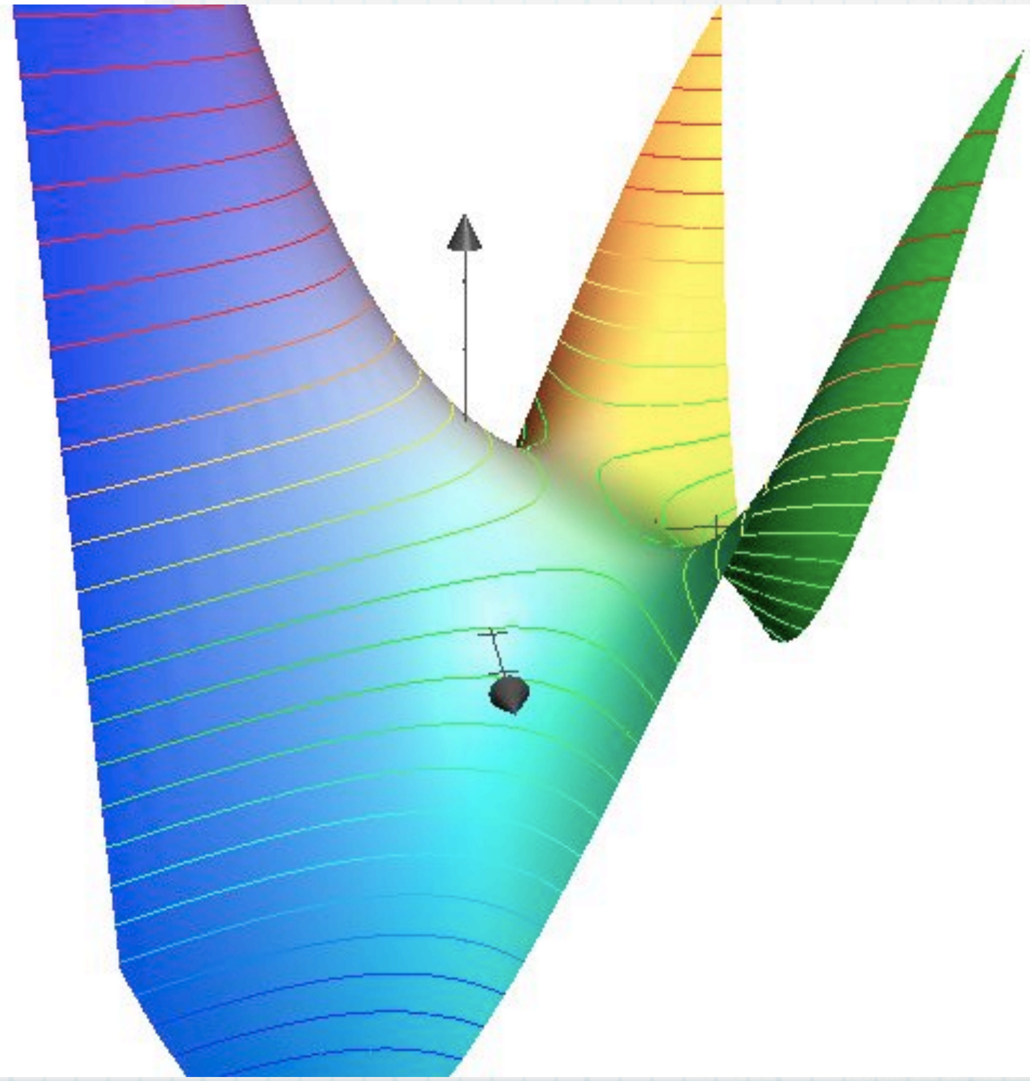
$$= 1 + x + \frac{1}{2}\left(x^2 - y^2\right) + \frac{1}{6}\left[x^3 \cdot 1 + 3x^2 y \cdot 0 + 3xy^2 \cdot (-1) + y^3 \cdot 0\right]$$

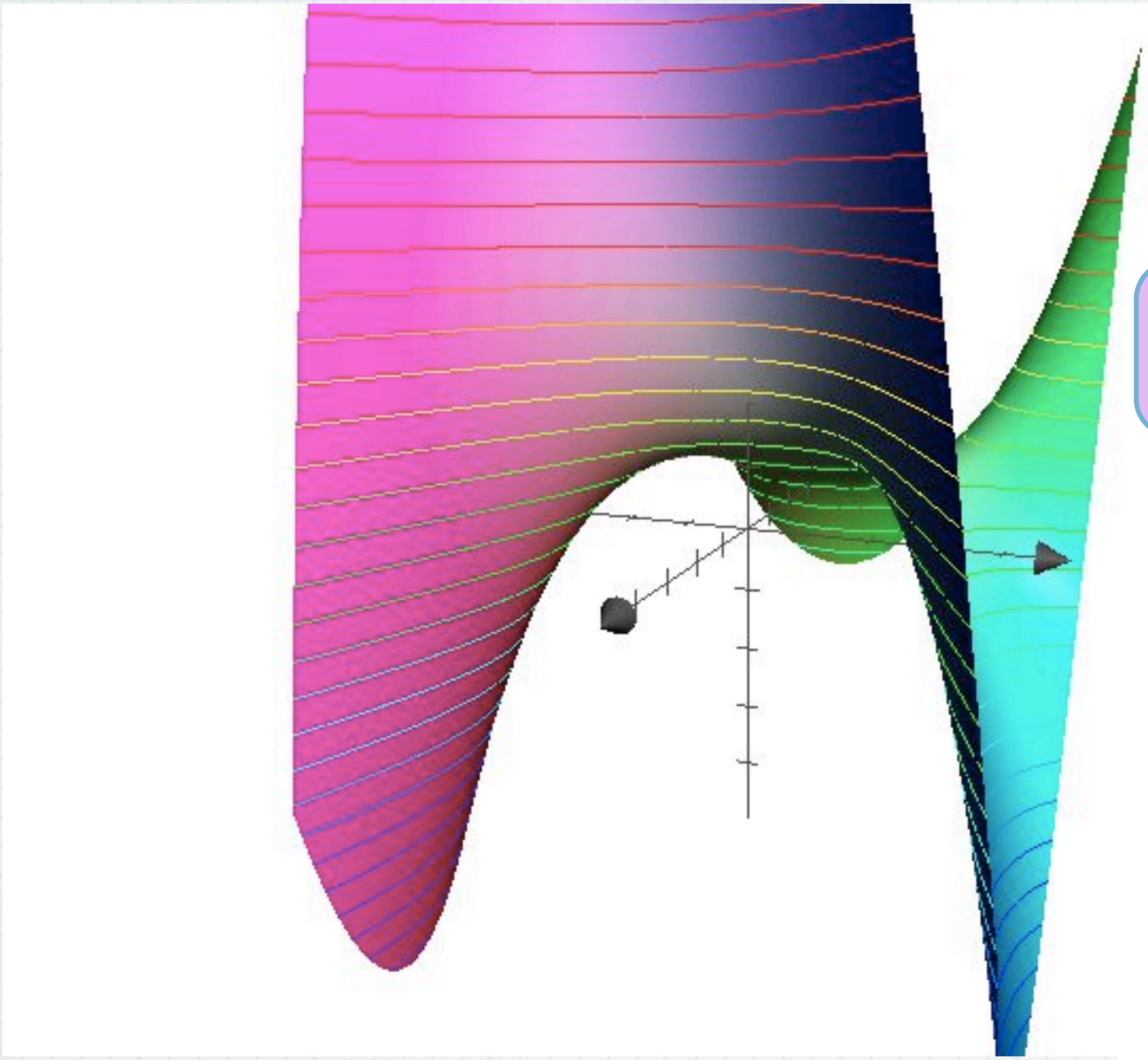
$$= \boxed{1 + x + \frac{1}{2}\left(x^2 - y^2\right) + \frac{1}{6}\left(x^3 - 3xy^2\right)}$$

Aproximación cubica

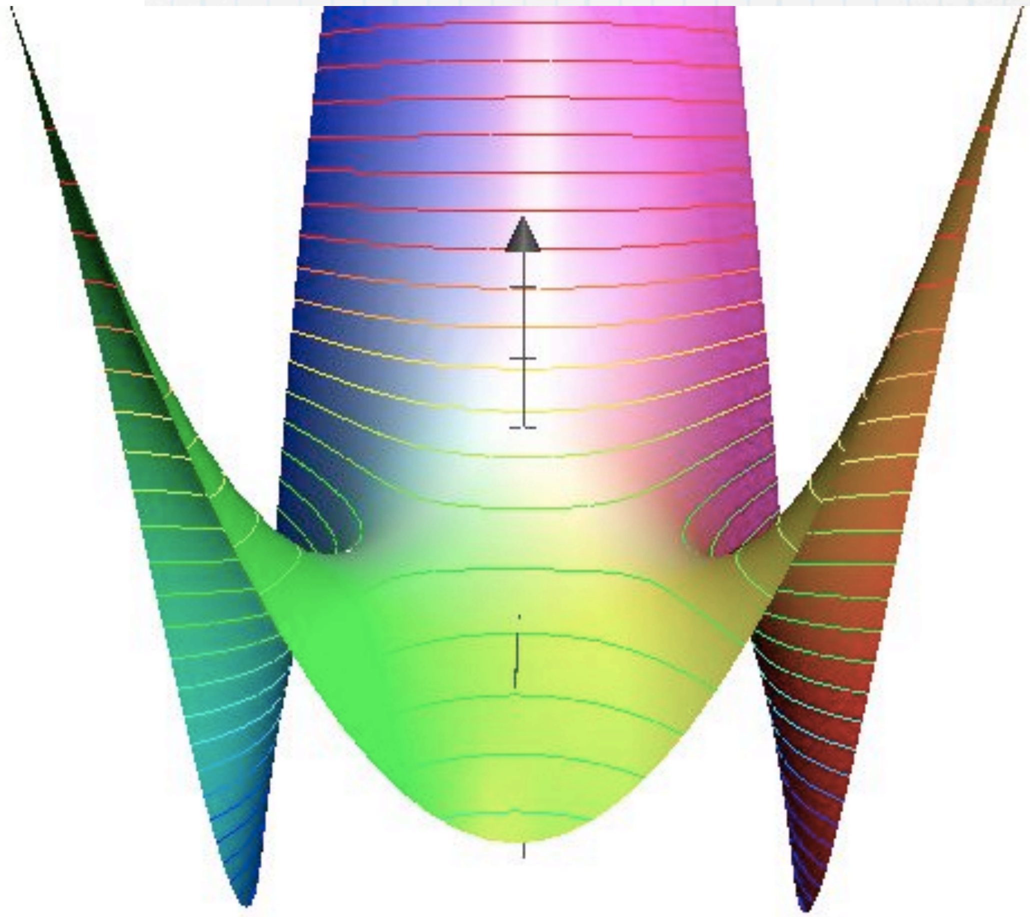


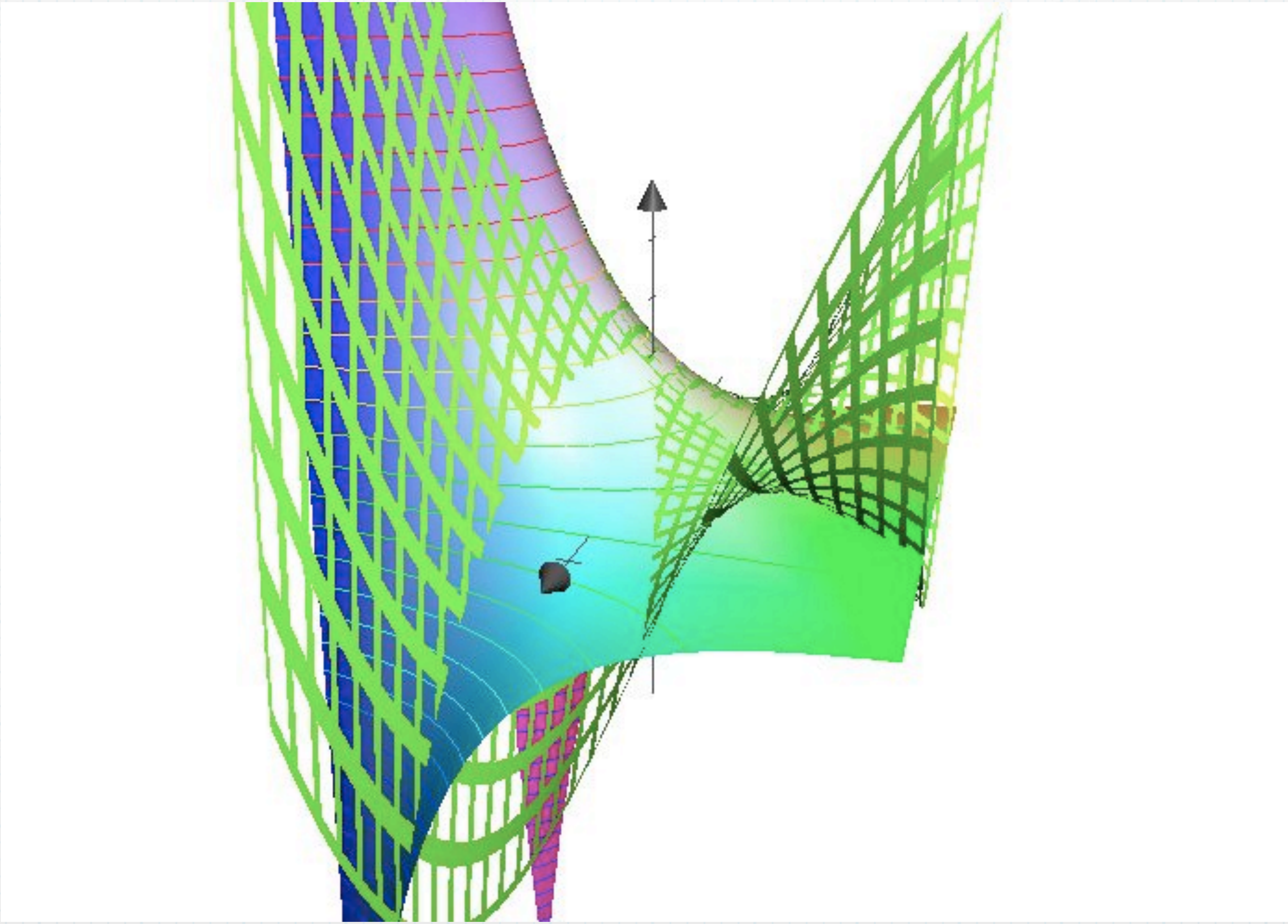
$$= 1 + x + \frac{1}{2}\left(x^2 - y^2\right) + \frac{1}{6}\left(x^3 - 3xy^2\right)$$





$$= 1 + x + \frac{1}{2}\left(x^2 - y^2\right) + \frac{1}{6}\left(x^3 - 3xy^2\right)$$





problema 2.

Utilice la formula de Taylor para encontrar una aproximación cuadratica de la función

$$f(x, y) = \cos(x)\cos(y)$$

en el origen. Estime el error en la aproximación si

$$|x| \leq 0,1 \text{ y } |y| \leq 0,1$$

$$f(x,y)=\cos x\cos y \;\Rightarrow\; f_x=-\sin x\cos y,$$

$$f_y=-\cos x\sin y,\; f_{xx}=-\cos x\cos y,\; f_{xy}=\sin x\sin y,$$

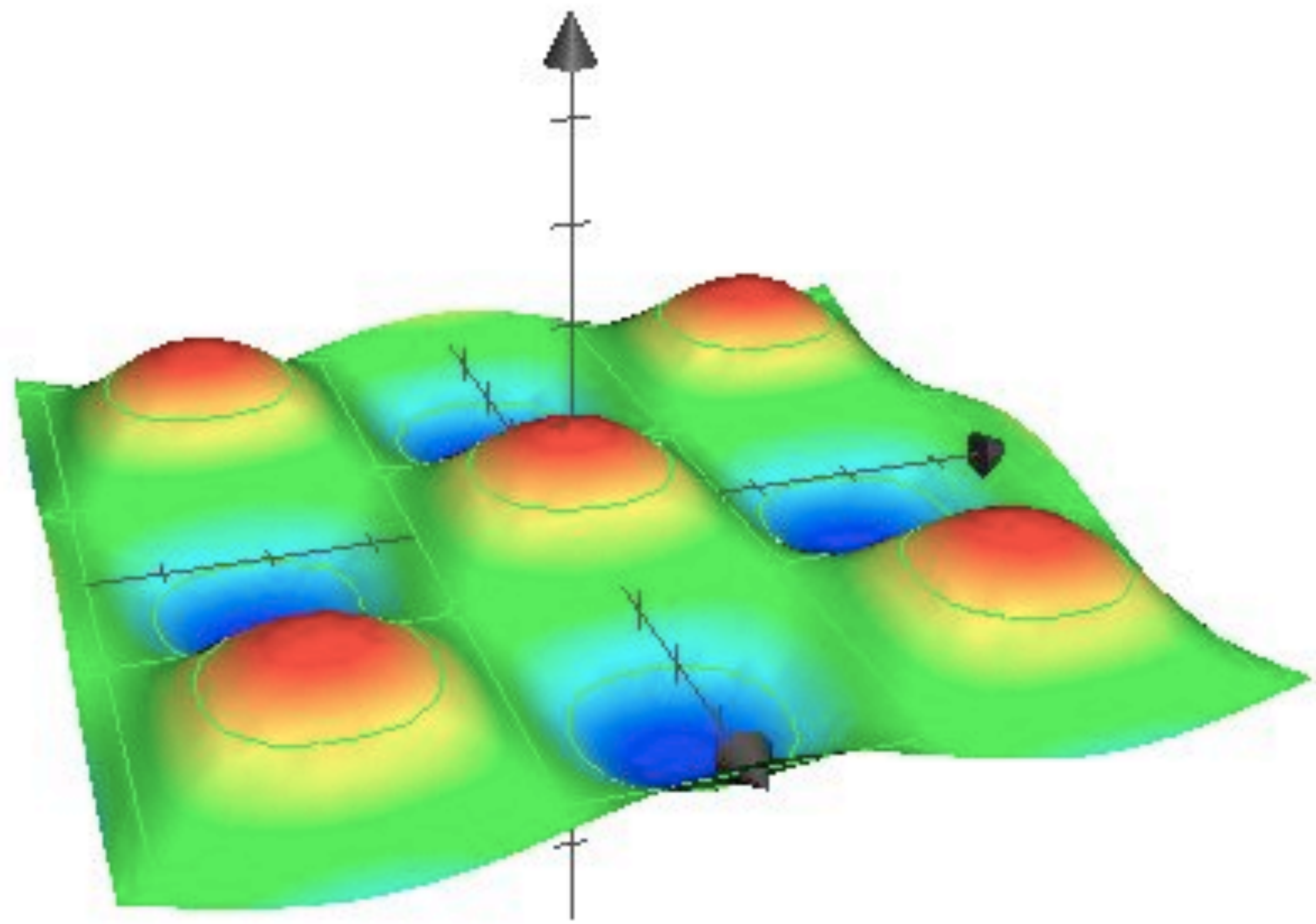
$$f_{yy}=-\cos x\cos y$$

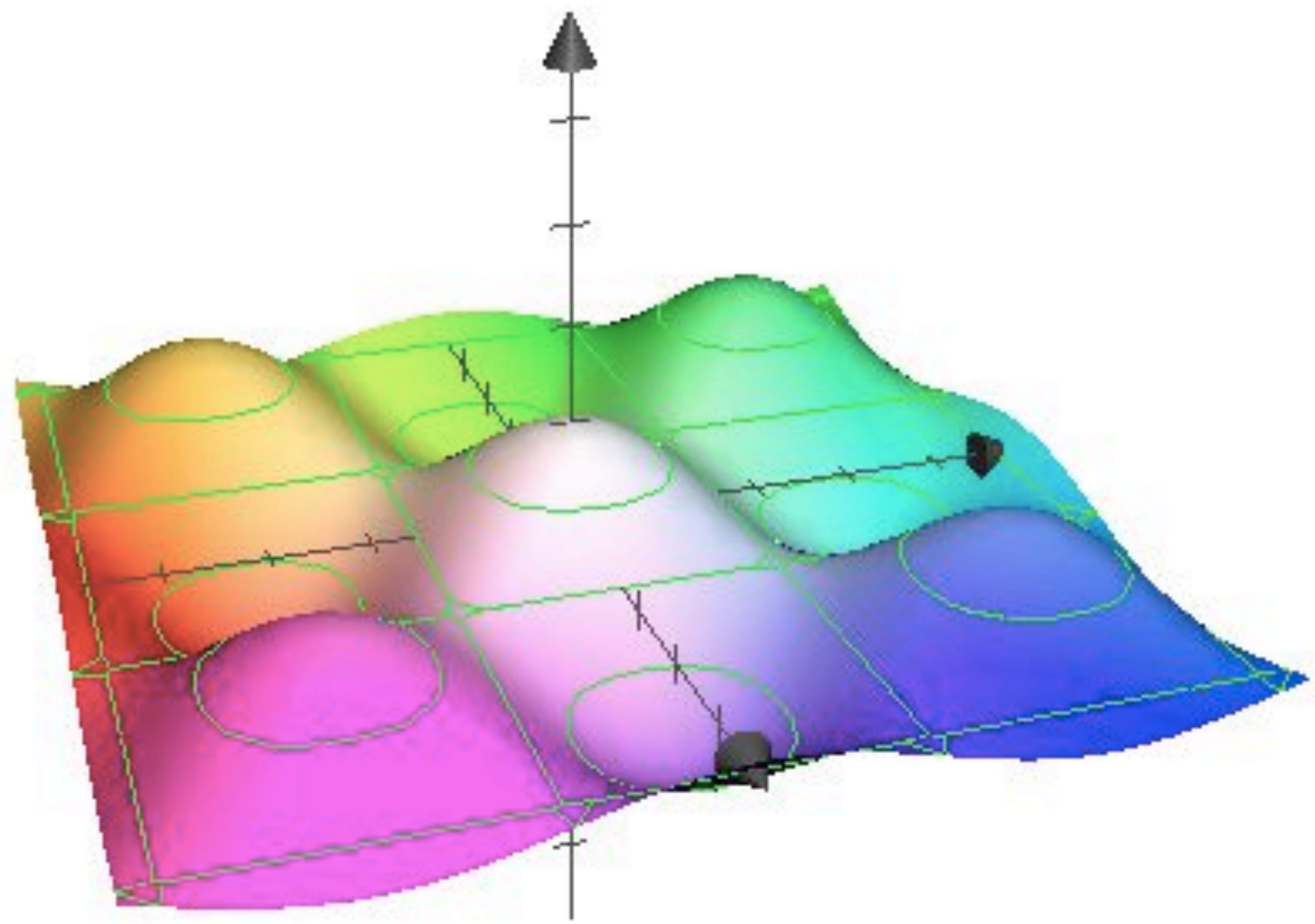
$$\Rightarrow\; f(x,y)\approx f(0,0)+xf_x(0,0)+yf_y(0,0)+\tfrac{1}{2}\left[x^2f_{xx}(0,0)+2xyf_{xy}(0,0)+y^2f_{yy}(0,0)\right]$$

$$=1+x\cdot 0+y\cdot 0+\tfrac{1}{2}\left[x^2\cdot(-1)+2xy\cdot 0+y^2\cdot(-1)\right]$$

$$=1-\frac{x^2}{2}-\frac{y^2}{2}$$

Aproximación cuadratica



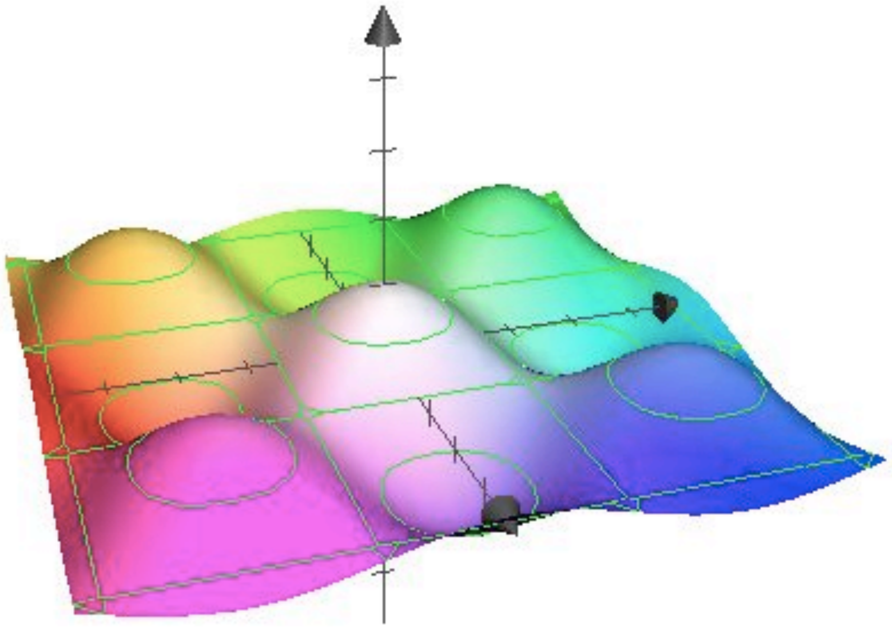


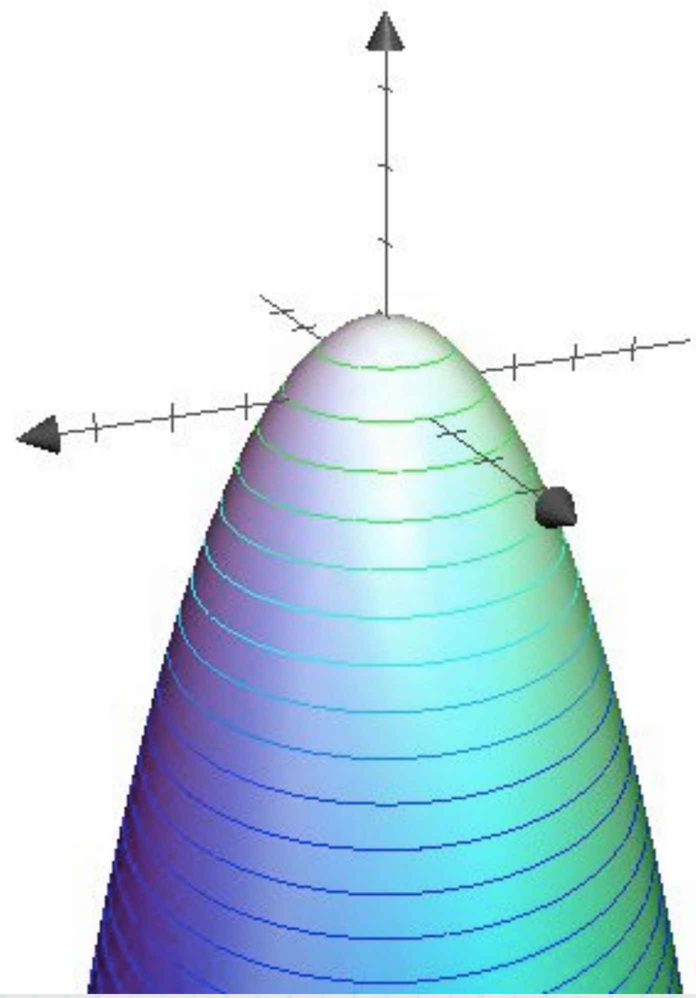
$$= 1 - \frac{x^2}{2} - \frac{y^2}{2}$$

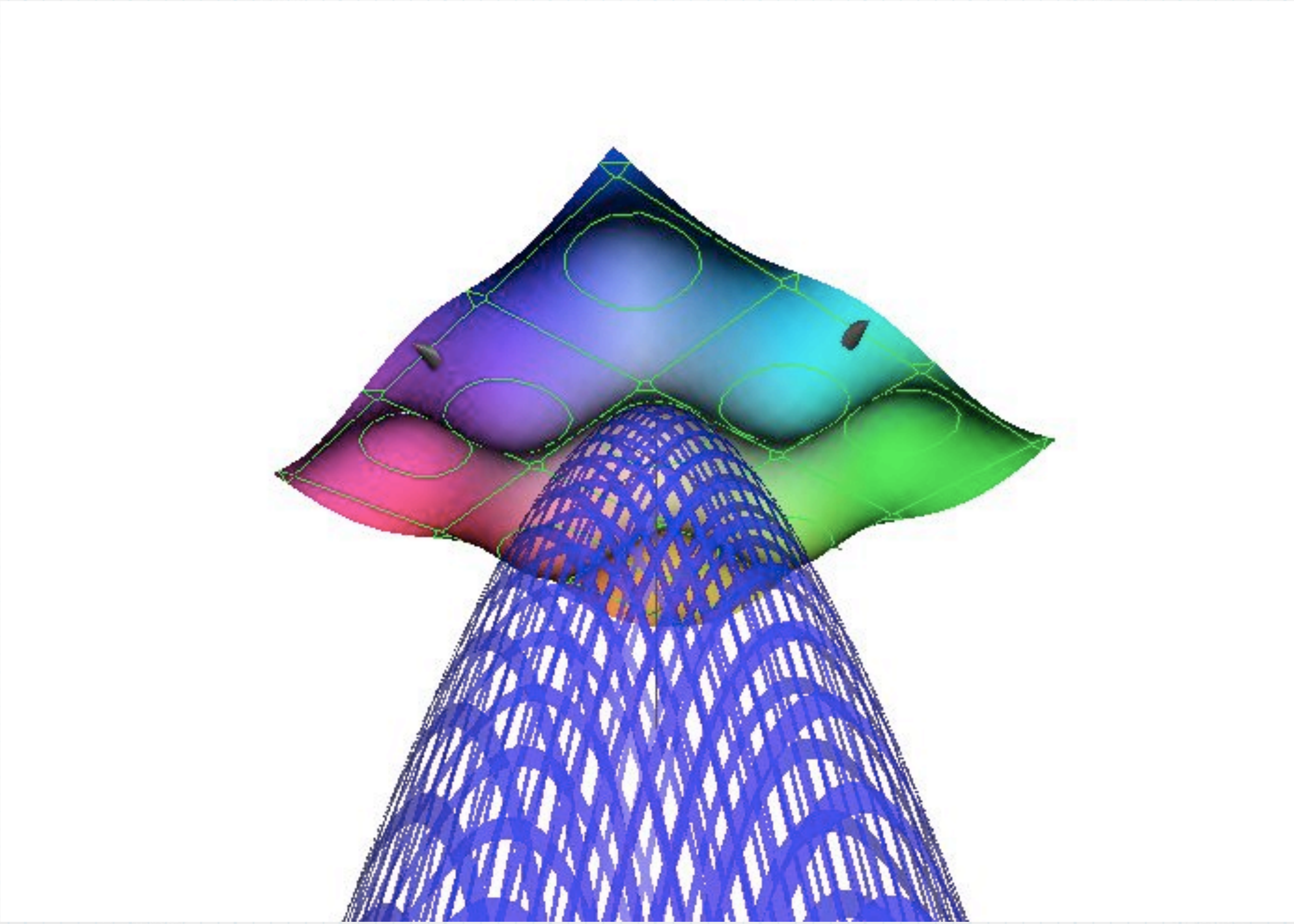
Aproximación cuadratica

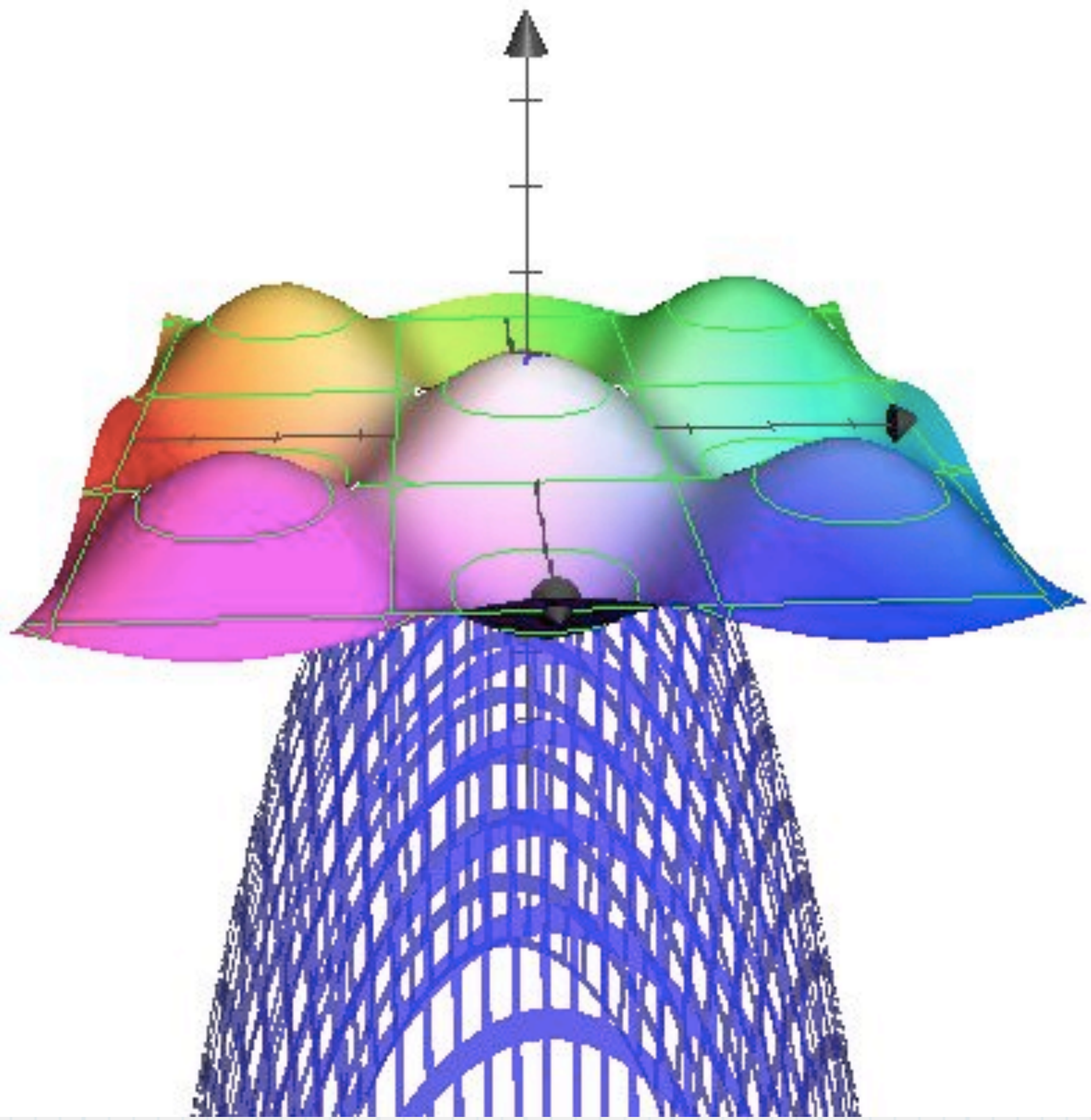
$$R(x, y) \leq \frac{1}{6}\left[(0,1)^3 + 3(0,1)^3 + 3(0,1)^3 + (0,1)^3\right]$$

$$R(x, y) \leq 0,00134$$









Ejercicio 2.

Repita el ejercicio anterior para la función

$$f(x,y) = e^{x}\mathrm{sen}(y)$$

r) $\mathrm{f(x,y) \approx y + xy}$

$$R(x,y) \leq 0,000814$$